

Lecture 5

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DATE 17-3-2016

vector analysis

التحليل المتجهي

$$\iiint_V (\nabla \cdot \vec{F}) dV = \oiint_S \vec{F} \cdot d\vec{S}$$

$$\oiint_S (\nabla \times \vec{F}) \cdot d\vec{S} = \oint_C \vec{F} \cdot d\vec{r}$$

If $\vec{H} = \nabla \times \vec{A}$ prove: $\oiint_S \vec{H} \cdot d\vec{S} = 0$
sol.

$$\oiint_S \vec{F} \cdot d\vec{S} = \iiint_V (\nabla \cdot \vec{F}) dV = \iiint_V (\nabla \cdot \nabla \times \vec{A}) dV$$

$$\oint_C \vec{E} \cdot d\vec{r} = -\frac{1}{c} \cdot \frac{\partial}{\partial t} \oiint_S \vec{H} \cdot d\vec{S}$$

$$\text{show } \nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{H}}{\partial t}$$

$$\oint_C \vec{E} \cdot d\vec{r} = \oiint_S \nabla \times \vec{E} \cdot d\vec{S}$$

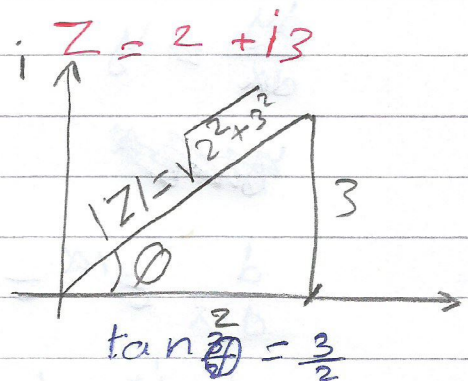
Complex analysis:-

التحليل المركب

Complex number:-

$$1 = \sqrt{1} \quad i = \sqrt{-1}$$

Z



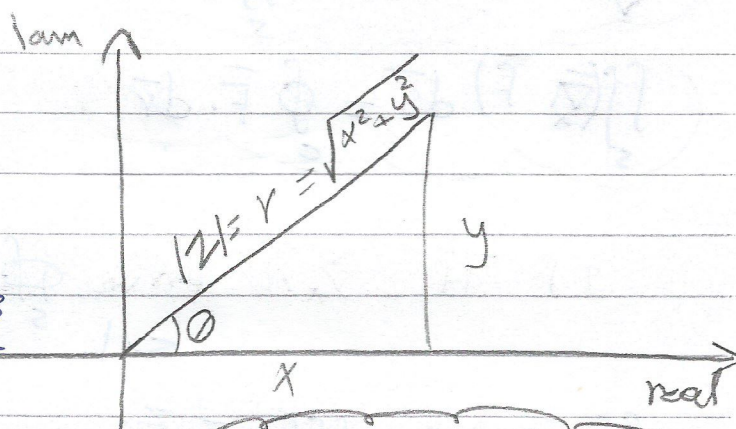
Complex variable

تغير المركب

$$Z = x + iy$$

$$Z = r \cos \theta + i r \sin \theta = x + iy$$

$$e^{ix} = 1 + \frac{ix}{1!} + \frac{-x^2}{2!} - \frac{i x^3}{3!} + \frac{x^4}{4!} + \dots$$



note: $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots$

① points to the exponential series in the cloud.
② points to the imaginary part of the Taylor series for e^{ix} .
③ points to the real part of the Taylor series for e^{ix} .

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$e^{ix} = \cos x + i \sin x$$

$$Z = r \cos \theta + i r \sin \theta = r e^{i\theta}$$

$$\frac{dy}{dx} = y$$

~~xxxxx~~

$$\frac{dy}{y} = dx$$

$$y = e^x$$

$$\frac{d}{dx} e^{ix} = e^{ix}$$

$$\frac{d}{dx} e^{ix} = i e^{ix}$$

$$\frac{d}{dx} (\cos x + i \sin x)$$

$$\begin{aligned} &= -\sin x + i \cos x \\ &= i (\cos x + i \sin x) \\ &= i (\cos x + i \sin x) \end{aligned}$$

$$\therefore e^{ix} = \cos x + i \sin x$$

$$\text{If } z = x + iy$$

$$f(z) = u(x, y) + i v(x, y) = u + i v$$

$$\text{If } z \rightarrow z_0 \quad \therefore x \rightarrow a_0, y \rightarrow b_0$$

$$f \rightarrow f_0, \quad u \rightarrow u_0, \quad v \rightarrow v_0$$

$$(\Delta z = \Delta x + i \Delta y)$$

ex.

$$f'(z) = \lim_{\Delta z \rightarrow 0} \frac{f(z + \Delta z) - f(z)}{\Delta z}$$

$$= \lim_{\Delta z \rightarrow 0} \frac{u(z + \Delta z) + i v(z + \Delta z) - [u(z) + i v(z)]}{\Delta z}$$

$$\text{If } \Delta z = \Delta x$$

$$= \lim_{\Delta x \rightarrow 0} \frac{u(x + \Delta x) + i v(x + \Delta x) - [u(x) + i v(x)]}{\Delta x}$$

$$= \left[\frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} \right] \rightarrow \textcircled{1}$$

$$\text{If } \Delta z = i \Delta y$$

$$f'(z) = \lim_{i \Delta y \rightarrow 0} \frac{u(x, y + \Delta y) + i v(x, y + \Delta y) - [u(x, y) + i v(x, y)]}{i \Delta y}$$

$$= \frac{1}{i} \frac{\partial u}{\partial y} + \frac{i}{i} \frac{\partial v}{\partial y} = \left[-i \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \right] \rightarrow \textcircled{2}$$

کے لیے $f(z)$ کا تعین

$$\therefore \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} = -i \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$$

$$\therefore \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

Cauchy - Riemann equation